Complexity of Judgement Aggregation

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The aggregation of individual preferences, judgements, or beliefs is a central problem in the study of rational interaction. Judgement aggregation focuses on sets of interconnected logical formulas (that we shall call the *agenda*) over which agents assign judgements of acceptance or rejection. The possibility of aggregating such judgements is undermined by a paradox, which was pointed out for the first time in a court case by Kornhauser and Sager (1986). The simplest *discursive dilemma*, as it has been named by Pettit (2001), works as follows: suppose a first judge accepts three propositions α, β and $\alpha \wedge \beta$, a second judge accepts only α and rejects the other two propositions, and similarly a third one accepts only β and rejects the others. Then majority voting as aggregation rule leads to an inconsistent outcome, since both α and β would be accepted, while $\alpha \wedge \beta$ would be rejected. Building from this example a number of results has been proved, leading to an independent and well-established subject (List and Puppe, 2009).

Our aim in this work is to establish the computational complexity of judgement aggregation, along similar lines as has been done very successfully in the literature on computational social choice for a number of related problems in voting theory (see e.g. Faliszewski et al., 2006). In particular, we will concentrate on the problem of determining whether consistent aggregation is possible on a given set of formulas, as a property of "safety" of the agenda under consideration. The complexity of this problem is important in a society of agents where a collective judgement has to be made frequently on different agendas. With this task comes the need of a precise and formally defined framework, and a discussion of the basic definitions from a computational point of view. Starting from the definition of aggregation function we will present in detail (the logical framework of aggregating sets of judgements). We will give our definition of discursive dilemma as a property of an aggregation function on a given agenda, and we will consider characterisation results for agendas that leads to inconsistent outcomes in this framework. Particular attention will be devoted to relate these results with the conditions and class restrictions already introduced in the literature (List and Puppe, 2009). We will then conclude by presenting some recent complexity results on the problem of determining if a certain class of aggregation functions generates a discursive dilemma with respect to a given agenda or a given profile of judgements. This is joint work with Ulle Endriss and Daniele Porello.

References

P. Faliszewski, E. Hemaspaandra, L. A. Hemaspaandra, and J. Rothe. A richer understanding of the complexity of election systems. *CoRR*, abs/cs/0609112, 2006.

L. A. Kornhauser and L. G. Sager. Unpacking the court. *The Yale Law Journal*, 96(1):82–117, 1986.

C. List and C. Puppe. Judgment aggregation: A survey. In *Handbook of Rational and Social Choice*. Oxford University Press, 2009.

P. Pettit. Deliberative democracy and the discursive dilemma. *Philosophical Issues (supplement to Nous)*, 11:268–99, 2001.