Linear Logic for Bidding Languages

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Abstract

Combinatorial auctions are auctions where bidders trade bundles of goods and they express their valuations over bundles by means of a bidding language that specifies the possible bids the auctioneer has to manage.

In this talk, we will present an application of linear logic as a language for multi-unit combinatorial auctions [1], in which the number of indistinguishable goods is relevant for the allocation procedure.

Linear logic is a resource-sensitive logic that provides tools to describe where the availability of hypotheses matters in a derivation, so we can use it to define bidding languages which can express in the syntax quantities of goods.

Simple bids will be interpreted as conditional formulas of linear logic stating, intuitively, that a bidder is disposed to pay a certain amount of money for certain bundles of goods.

Then, we will see types of bidding languages to define complex bids that resemble the usual languages for combinatorial auctions (OR, XOR [4] and Goal Base languages [5]), obtained considering fragments of the language of linear logic.

The syntax of linear logic allows, for example, to express in a precise way some interesting distinctions between types of goods, such as reusable and non-reusable goods or sharable and non-sharable goods.

We will interpret the procedure of allocation, and the winner determination problem, by means of proof search in a sequent calculus for linear logic. For example, if we consider Horn fragments of linear logic [3], we have proof search in NP.

At the end, we will make some remarks on a sort of generalized auction, in which agents trade any kind of formula and the provability of the sequent containing them represents a form of agreement between sellers and buyers. This is joint work with Ulle Endriss.

References

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